

MAS 117

Introduction to Algebra

Assignment 2

For handing in on 24 January 2007

*Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the **RED** post-box on the **BASEMENT** of the Maths building before 16:00 on Wednesday.*

1 (30 marks) Use induction to prove each of the following statements:

(a) For all $n \geq 1$,

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{(2n-1) \times (2n+1)} = \frac{n}{2n+1}.$$

(b) $4^n \geq 16n^2$ for all $n \geq 4$.

2 (25 marks) Here is an extract from the Geometry I lecture notes. I've written vectors in bold, instead of underlined (thus, **v** instead of v).

The scalar product is a way of multiplying two vectors to produce a scalar (a real number).

Let **u**, **v** be non-zero vectors represented by $\vec{AB}$, $\vec{AC}$. We define the *angle* between **u** and **v** to be the angle θ (in radians), with $0 \leq \theta \leq \pi$, between $\vec{AB}$ and $\vec{AC}$.

The *scalar product* (or *dot product*) **u** · **v** is $|\mathbf{u}| |\mathbf{v}| \cos \theta$, if **u** and **v** are non-zero vectors and θ is the angle between them. If **u** = **0** or **v** = **0** then we define **u** · **v** = 0.

- (a) What is this? Is it a definition, a theorem, a proof, an example, or just some chit-chat?
- (b) Your boss asks you to summarise it in a brief bullet point, without using any symbols. What would you say?
- (c) Why are the zeros in the last sentence written in different typefaces?

3 (20 marks) Use the definition of multiplication for complex numbers to verify that they satisfy the commutative and associative laws.

4 (25 marks) Factorise the polynomial $x^4 + 4$ into two quadratics with real coefficients.

5 Prize question A cash prize is offered for the best solution received to this question. Do not hand this in with the rest of your coursework; give your solutions to me.

The *quaternions* are expressions of the form $a + bi + cj + dk$, where a, b, c, d are real numbers. They are added by the “obvious rule”, and multiplication is based on the formulae

$$i^2 = j^2 = k^2 = ijk = -1.$$

- (a) Prove that the associative law for multiplication holds.
- (b) Prove that the distributive law holds.
- (c) Prove that the commutative law for multiplication does not hold.
- (d) Prove that

$$(a + bi + cj + dk)(a - bi - cj - dk) = (a^2 + b^2 + c^2 + d^2)$$

and hence show that every non-zero quaternion has a multiplicative inverse.